BC Calculus Quiz #11 v.A
Integration Stuff
Dr. Wisniewski Spring 2020

Name Solution de Baĝbabii **Period** ooti

Instructions: Solve each of the problems below. Please show your work (for partial credit) and box or circle your answers. A calculator is NOT permitted on this quiz.

1. (2 Pts) Let $f(x) = \int_{-5}^{x^2} \frac{1}{t^2+1}\,dt$. Find $f'(2)$.

$$f'(x) = \frac{1}{(x^2)^2+1} \cdot 2x = \frac{2x}{x^4+1}$$

$$f'(2) = \frac{2\cdot 2}{2^4+1} = \frac{4}{16+1} = \boxed{4/17}$$

2. (4 Pts) Find the average value of the function $f(x) = 16 - x^2$ on $[-4, 4]$. Find at least one value of x, call it $x = c$, for which $f(c) =$ the average value.

$$f_{avg} = f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-(-4)}\int_{-4}^{4} 16-x^2 = \frac{2}{8}\int_0^4 16-x^2$$

$$f_{avg} = \frac{1}{4}\left[16x - \frac{x^3}{3}\right]_0^4 = \frac{1}{4}\left[16\cdot 4 - \frac{4^3}{3}\right] = \frac{1}{4}\left[64 - \frac{64}{3}\right] = \frac{1}{4}\left(\frac{192-64}{3}\right)$$

$$= \frac{128}{12} = \frac{64}{6} = \frac{32}{3}$$

$$\boxed{f_{avg} = 32/3}$$

$$f(c) = 16 - c^2 = \frac{32}{3}$$

$$16 - \frac{32}{3} = c^2$$

$$\frac{48-32}{3} = c^2$$

$$\frac{16}{3} = c^2$$

$$\boxed{c = \pm\frac{4}{\sqrt{3}}} \pm\frac{4\sqrt{3}}{3}$$

3. (2 Pts) The average value of a continuous function $f(x)$ on $[3, 7]$ is 12. What is the value of $\int_3^7 f(x)dx$?

$$f_{avg} = 12 = \frac{1}{7-3}\int_3^7 f(x)\,dx = \frac{1}{4}\int_3^7 f(x)\,dx$$

$$\boxed{48 = \int_3^7 f(x)\,dx}$$

4. (4 Pts) A large tank initially contains 132 gallons chemical X. Starting at $t = 0$, where t is measured in minutes, more chem X is pumped into the tank at the rate $3t^2 + 1$ gallons per minute. What volume of chem X is in the tank at $t = 5$ minutes?

$V(5) - V(0) = \int_0^5 (3t^2+1)\,dt$ ← $V'(t)$

$V(5) = 132 + \int_0^5 (3t^2+1)\,dt = 132 + \left[t^3 + t \right]_0^5$

$V(5) = 132 + 125 + 5 =$ **262 gallons**

-.5 for no units!

5. Evaluate each of the following integrals.

Spikes

a. $\int_{-\pi/4}^{\pi/4} \tan x\,dx = 0$ odd funct

(+1)

b. $\int \frac{x^2+5x-1}{x}\,dx = \int \left(x + 5 - \frac{1}{x}\right) dx = \frac{x^2}{2} + 5x - \ln|x| + C$

(+1)

c. $\frac{d}{dt}\int_{-2}^{8} e^x\,dx = 0$ deriv of a konstant!

26/30 students got this correct. good!

(-1) if u R one of the four who didn't — there's no reason for u 2 B not getting this correct — plug in !! its like the 82nd time I've done this! the deriv of a const = 0 !!!!!!!!!

(-2) d. $\int_{-1}^{4} |3x-2|\,dx$

$3x-2=0$

$3x=2$

$x=2/3$

$\int_{-1}^{4} |3x-2|\,dx = \int_{-1}^{2/3} (2-3x)\,dx + \int_{2/3}^{4} (3x-2)\,dx = \left[2x - \frac{3x^2}{2}\right]_{-1}^{2/3} + \left[\frac{3x^2}{2} - 2x\right]_{2/3}^{4}$

$= \frac{4}{3} - \frac{3}{2}\cdot\frac{4}{9} - (-2-3/2) + \left[24 - 8 - \left(\frac{3}{2}\cdot\frac{4}{9} - \frac{4}{3}\right)\right]$

$= \frac{4}{3} - \frac{2}{3} + \frac{7}{2} + 16 - \left(\frac{2}{3} - \frac{4}{3}\right) = \frac{3}{2} + 16 + \frac{2}{3} = \frac{125}{6}$ ✓

$= \frac{2}{3} + \frac{7}{2} - 3 = 25/6$

$\frac{25}{2} + 100$

BC Calculus Quiz #11 v.B

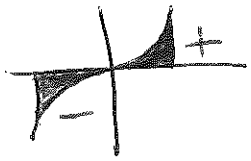
Integration Stuff Dr. Wisniewski Spring 2020

Name Solution **Period** 2

Instructions: Solve each of the problems below. Please show your work (for partial credit) and box or circle your answers. A calculator is NOT permitted on this quiz.

1. Evaluate each of the following integrals.

(+1) a. $\int_{-\pi/4}^{\pi/4} \tan x \, dx$ $= 0$ $y = \tan x$ is odd funct.



5 pts

(+1) b. $\int \frac{x^2+5x-1}{x} dx$ $= \int (x + 5 - \frac{1}{x}) dx = \boxed{\frac{x^2}{2} + 5x - \ln|x| + C}$

(+1) c. $\frac{d}{dt}\int_{-2}^{8} e^x dx = 0$ deriv. of a konstant $= 0$

area under curve $= \#$

(+2) d. $\int_{-1}^{4} |3x - 2| \, dx$ $3x - 2 = 0 \to 3x = 2 \to x = 2/3$

$$\int_{-1}^{4} |3x-2| dx = \int_{-1}^{2/3} -(3x-2) dx + \int_{2/3}^{4} (3x-2) dx$$

$$= \left[\frac{3x^2}{2} - 2x \right]_{2/3}^{-1} + \left[\frac{3x^2}{2} - 2x \right]_{2/3}^{4}$$

$$= \frac{3}{2} + 2 \cdot 1 - \left(\frac{3}{2} \cdot \left(\frac{2}{3}\right)^2 - \frac{4}{3} \right) + \frac{3}{2} \cdot 16 - 8 - \left(\frac{3}{2} \cdot \left(\frac{2}{3}\right)^2 - \frac{4}{3} \right)$$

$$= \frac{7}{2} - \left(-\frac{2}{3}\right) + 16 - \left(-\frac{2}{3}\right)$$

$$= \frac{7}{2} + \frac{4}{3} + 16 = \frac{21 + 8 + 96}{6} = \boxed{\frac{125}{6}}$$

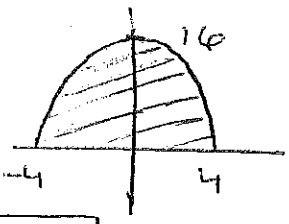
2. (2 Pts) Let $f(x) = \int_{-2}^{x^2} \frac{1}{t^2+1} dt$. Find $f'(3)$.

$$f'(x) = \frac{1}{(x^2)^2 + 1} \cdot 2x = \frac{2x}{x^4 + 1} \qquad f'(3) = \frac{2 \cdot 3}{3^4 + 1} = \frac{6}{82} = \frac{3}{41}$$

3. (4 Pts) Find the average value of the function $f(x) = 16 - x^2$ on $[-4, 4]$. Find at least one value of x, call it $x = c$, for which $f(c) =$ the average value.

$f_{avg} = f(c) = \frac{1}{4-(-4)} \int_{-4}^{4} (16 - x^2)\,dx = \frac{2}{8} \int_{0}^{4} (16 - x^2)\,dx$

$= \frac{1}{4}\left[16x - \frac{x^3}{3}\right]_0^4 = \frac{1}{4}\left(64 - \frac{64}{3}\right) = \frac{1}{4}\left(\frac{192-64}{3}\right) = \frac{128}{12}$



$\boxed{f_{avg} = \frac{32}{3}}$

$f(c) = 16 - c^2 = \frac{32}{3}$

$16 - \frac{32}{3} = c^2$

$\frac{48-32}{3} = \frac{16}{3} = c^2$

$\boxed{c = \pm \frac{4}{\sqrt{3}}}$

192
−64
128

4. (2 Pts) The average value of a continuous function $f(x)$ on $[3, 7]$ is 20. What is the value of $\int_3^7 f(x)dx$?

$f(c) = 20 = \frac{1}{4} \int_3^7 f(x)\,dx$

$\boxed{80 = \int_3^7 f(x)\,dx}$

5. (4 Pts) A large tank initially contains 127 gallons chemical X. Starting at $t = 0$, where t is measured in minutes, more chem X is pumped into the tank at the rate $3t^2 + 2$ gallons per minute. What volume of chem X is in the tank at $t = 5$ minutes?

$V(t) =$ Vol. of chem X in tank at time t

$V(5) - V(0) = \int_0^5 V'(t)\,dt$

$V(5) = \underbrace{V(0)}_{127} + \int_0^5 (3t^2 + 2)\,dt = 127 + \left[t^3 + 2t\right]_0^5$

$V(5) = 127 + \left[5^3 + 10\right] = 127 + 135 = 262$ gal

135
127
262

$\boxed{V(5) = 262 \text{ gallons}}$

-.5 for no units.